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Quantum-mechanical decay laws in the neutral kaons

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Abstract
The Hamiltonian Friedrichs model [1] describing the evolution of a two-level system coupled to a continuum is used in order to modelize the decay of the kaon states K_1, K_2. Using different cut-off functions of the continuous degrees of freedom, we show that this model leads to a CP violation that qualitatively fits with experimental data improving previous numerical estimates. We also discuss the relation of our model to other models of open systems.

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Introduction

There have been several theoretical approaches to CP violation in kaons (see e.g, the collection of papers edited in [2]), and the question is partially open today. In this paper, we use a Hamiltonian model, describing a two-level states coupled to a continuum of degrees of freedom, that makes it possible to obtain the phenomenology of neutral kaons and provides a new numerical estimate of the parameters of CP violation. Solving the Schrödinger equation for the Hamiltonian, we derive a master equation for the decaying two-level states similarly to the generalization of the Weisskopf–Wigner approach formulated by Lee, Oeheme and Yang [3] (LOY) in the case of kaonic decay. Later on, Chiu and Sudarshan [4] used a Lee model in order to obtain a correction to the LOY theory for short times, departing from the exponential decay. Their Hamiltonian describes $(K^0, \overline{K}^0)$ modes as done in the LOY theory. In this paper the authors point out to a numerical Khalfin estimate of the CP violation parameter $\epsilon \sim 0.06 \times e^{i\pi/4}$. That is 30 times the experimental data. Our new approach is based on the derivation of a master equation from a Hamiltonian describing (K_1, K_2) decaying modes and not for $(K^0, \overline{K}^0)$ modes as done in LOY theory. Under weak coupling hypothesis this leads to a Markovian master equation which allows us to simulate the kaonic lifetimes as well as kaonic oscillations and regeneration. It even fits closer the CP symmetry breaking parameter.

Unfortunately, this last prediction is not quite accurate quantitatively, which, in a sense, is not astonishing for such a simplified approach. In a first example with non-bounded spectrum in energy, we obtain the exact angle while the modulus is 14 times the experimental data. However, we show that using different cut-off functions of the continuous degrees of freedom we can improve the above estimate.

We prove that it is possible to obtain all the interesting features of the model when the Hamiltonian possesses a spectrum only bounded from below. In this case, with Gaussian cut-off the previous estimate is improved and we obtain a CP violation parameter value only three times the experimental data. Our treatment confirms that it is possible with a very simple model such as the two-level Friedrichs model to compute some essential features of the very rich kaon phenomenology. It also confirms that the essential ingredient for deriving an irreversible in time dynamics of subsystems is the presence of a continuous degrees of freedom of environment.

In general, quantum mechanics provides a continuous, reversible in time and unitary evolution law (via the Schrödinger equation). This description contradicts our everyday experience in which ageing, dissipation and irreversibility are omnipresent. In this context, it is interesting to study hybrid quantum systems, sufficiently complex, that exhibit altogether unitary and dissipative in time evolutions. This goal can be achieved in the framework of the Friedrichs model.

One-level Friedrichs model is well understood [5–7]: it predicts that the excited state disappears and 'fuses' into the continuum. Its survival probability decays exponentially in time. The lifetime is proportional to the coupling between the discrete mode and the continuum. Exponentially decaying systems are very common in classical and quantum physics. They are relatively trivial when we consider them from the point of view of temporal irreversibility because, although the decay law is not reversible in time, such systems behave as if they did not possess an internal clock or memory: the decay rate is constant throughout time, and the non-decayed system is in the same state at all times. Roughly speaking, exponentially decaying systems exhibit an irreversible behaviour but ignore ageing.

The two-level Friedrichs system makes it possible to describe a class of systems that exhibit richer behaviour: oscillations, regenerations and so on. If we accept a general definition [8, 9] according to which each departure from the pure exponential decay law can be labelled as a Zeno behaviour (or anti-Zeno, depending on the sign of the departure), then, as we shall show the two-level Friedrichs model is rich enough in order to describe Zeno and anti-Zeno behaviour (for N-levels generalizations see [9]), and provides a relatively exact phenomenological model of kaons physics.

In the first section we define the two-level Friedrichs model. In the second section we recall the main features of kaon phenomenology. In the third section we show how to simulate them, thanks to the Friedrichs model, when (a) the spectrum of the continuous mode is unbounded, in the presence of a Gaussian cut-off and (b) in the presence of a Gaussian cut-off when negative energy levels of the continuum are decoupled from the two-level system.

In the fourth section we compare our approach with other recent approaches [10–17] where an open system interacts with its environment having a Lindblad form of evolution, and we also discuss the question of decoherence.

1. The two-level Friedrichs model

The Friedrichs interaction Hamiltonian between the two discrete modes and the continuous degree of freedom is given by the operator H on the Hilbert space of the wavefunctions of the

form $|\psi\rangle = \{f_1, f_2, g(\omega, t)\}$, $f_1, f_2 \in \mathbb{C}$, $g \in L^2(\mathbb{R}^+)$:

$$H = H_0 + \lambda_1 V_1 + \lambda_2 V_2, \tag{1.1}$$

where λ_1 and λ_2 are the positive coupling constants, and

$$H_0|\psi\rangle = \{\omega_1 f_1, \omega_2 f_2, \omega g(\omega, t)\}, (\omega_1 \text{ and } \omega_2 > 0). \tag{1.2}$$

The operators $V_i (i = 1, 2)$ are given by

$$\begin{aligned} V_1\{f_1, f_2, g(\omega, t)\} &= \{\langle v(\omega), g(\omega, t)\rangle, 0, f_1.v(\omega)\} \\ V_2\{f_1, f_2, g(\omega, t)\} &= \{0, \langle v(\omega), g(\omega, t)\rangle, f_2.v(\omega)\}, \end{aligned} \tag{1.3}$$

where

$$\langle v(\omega), g(\omega, t)\rangle = \int \mathrm{d}\omega\, v^*(\omega) g(\omega, t) \tag{1.4}$$

is the inner product. Thus H can be represented as a matrix:

$$H_{\text{Friedrich}} = \begin{pmatrix} \omega_1 & 0 & \lambda_1 v^*(\omega) \\ 0 & \omega_2 & \lambda_2 v^*(\omega) \\ \lambda_1 v(\omega) & \lambda_2 v(\omega) & \omega \end{pmatrix}. \tag{1.5}$$

Here $\omega_{1,2}$ represent the energies of the discrete levels and the factors $\lambda_i v(\omega) (i = 1, 2)$ represent the couplings to the continuous degree of freedom. The energies ω of the different modes of the continuum range from $-\infty$ to $+\infty$ when $v(\omega) = 1$, but we are free to tune the coupling $v(\omega)$ in order to introduce a selective cut-off to extreme energy modes. Let us now solve the Schrödinger equation and trace out the continuum in order to derive the master equation for the two-level system. The two-level Friedrichs model Schrödinger equation with $\hbar = 1$ is formally written as

$$\begin{pmatrix} \omega_1 & 0 & \lambda_1 v^*(\omega) \\ 0 & \omega_2 & \lambda_2 v^*(\omega) \\ \lambda_1 v(\omega) & \lambda_2 v(\omega) & \omega \end{pmatrix} \begin{pmatrix} f_1(t) \\ f_2(t) \\ g(\omega, t) \end{pmatrix} = \mathrm{i}\frac{\partial}{\partial t} \begin{pmatrix} f_1(t) \\ f_2(t) \\ g(\omega, t) \end{pmatrix}. \tag{1.6}$$

Easily, we obtain

$$\omega_1 f_1(t) + \lambda_1 \int \mathrm{d}\omega\, v^*(\omega) g(\omega, t) = \mathrm{i}\frac{\partial f_1(t)}{\partial t}, \tag{1.7}$$

$$\omega_2 f_2(t) + \lambda_2 \int \mathrm{d}\omega\, v^*(\omega) g(\omega, t) = \mathrm{i}\frac{\partial f_2(t)}{\partial t}, \tag{1.8}$$

and

$$\lambda_1 v(\omega) f_1(t) + \lambda_2 v(\omega) f_2(t) + \omega g(\omega, t) = \mathrm{i}\frac{\partial g(\omega, t)}{\partial t}. \tag{1.9}$$

Integrating the last equation we obtain $g(\omega, t)$ assuming $g(\omega, t = 0) = 0$:

$$g(\omega, t) = -\mathrm{i}\,\mathrm{e}^{-\mathrm{i}\omega t} \int_0^t \mathrm{d}\tau [\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)] v(\omega)\, \mathrm{e}^{\mathrm{i}\omega\tau}, \tag{1.10}$$

then, we substitute $g(\omega, t)$ in the above equation (1.7) and obtain

$$\mathrm{i}\frac{\partial f_1(t)}{\partial t} = \omega_1 f_1(t) - \mathrm{i}\lambda_1 \int \mathrm{d}\omega |v(\omega)|^2\, \mathrm{e}^{-\mathrm{i}\omega t} \int_0^t \mathrm{d}\tau [\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)]\, \mathrm{e}^{\mathrm{i}\omega\tau}, \tag{1.11}$$

we also obtain the same relation for $f_2(t)$ as

$$\mathrm{i}\frac{\partial f_2(t)}{\partial t} = \omega_2 f_2(t) - \mathrm{i}\lambda_2 \int \mathrm{d}\omega |v(\omega)|^2\, \mathrm{e}^{-\mathrm{i}\omega t} \int_0^t \mathrm{d}\tau [\lambda_1 f_1(\tau) + \lambda_2 f_2(\tau)]\, \mathrm{e}^{\mathrm{i}\omega\tau}. \tag{1.12}$$

In the following section, we shall make use of the Friedrichs model in order to simulate interesting properties of the kaonic systems. Before we do so, it is useful to recall certain of them.

2. Main features of kaon phenomenology

Kaons are bosons that were discovered in the 1940s during the study of cosmic rays. They are produced by collision processes in nuclear reactions during which the strong interactions dominate. They appear [18, 19] in pairs $\mathrm{K}^0, \overline{\mathrm{K}}^0$. It is possible to produce preferentially the K^0 particle essentially due to the fact that the $\overline{\mathrm{K}}^0$ kaon is less probable kinematically and that the threshold pion energy for its production is higher.

The K mesons are eigenstates of the parity operator P: $P|\mathrm{K}^0\rangle = -|\mathrm{K}^0\rangle$ and $P|\overline{\mathrm{K}}^0\rangle = -|\overline{\mathrm{K}}^0\rangle$. K^0 and $\overline{\mathrm{K}}^0$ are charge conjugate to each other $C|\mathrm{K}^0\rangle = |\overline{\mathrm{K}}^0\rangle$ and $C|\overline{\mathrm{K}}^0\rangle = |\mathrm{K}^0\rangle$. We thus get

$$CP|\mathrm{K}^0\rangle = -|\overline{\mathrm{K}}^0\rangle, \qquad CP|\overline{\mathrm{K}}^0\rangle = -|\mathrm{K}^0\rangle. \tag{2.1}$$

Clearly $|\mathrm{K}^0\rangle$ and $|\overline{\mathrm{K}}^0\rangle$ are not CP eigenstates, but the following combinations,

$$|\mathrm{K}_1\rangle = \frac{1}{\sqrt{2}}(|\mathrm{K}^0\rangle + |\overline{\mathrm{K}}^0\rangle), \qquad |\mathrm{K}_2\rangle = \frac{1}{\sqrt{2}}(|\mathrm{K}^0\rangle - |\overline{\mathrm{K}}^0\rangle), \tag{2.2}$$

are CP eigenstates.

$$CP|\mathrm{K}_1\rangle = -|\mathrm{K}_1\rangle, \qquad CP|\mathrm{K}_2\rangle = +|\mathrm{K}_2\rangle. \tag{2.3}$$

In the absence of matter, kaons disintegrate through weak interactions. Actually, K^0 and $\overline{\mathrm{K}}^0$ are distinguished by their mode of *production*, and K_1 and K_2 are distinguished by their mode of *decay* [18]. In first approximation, we can neglect CP violation so that the weak Hamiltonian commutes with CP. In this regime, the weak disintegration process distinguishes the K_1 and the K_2 states. The lifetime of the K_1 kaon is short $\left(\tau_S \approx 8.92 \times 10^{-11}\ \mathrm{s}\right)$, while the lifetime of the K_2 kaon is longer ($\tau_L \approx 5.17 \times 10^{-8}$ s). The difference of the mass of the 1 and 2 kaons is quite small in comparison to their mass ($\frac{m_L - m_S}{m_S + m_L} \approx 0.35 \times 10^{-14}$, with $(m_L - m_S)c^2 \approx 3.52 \times 10^{-6}$ eV). The amplitudes of state K_1 at time t can be written as

$$a_1(t) = a_1(0)\,\mathrm{e}^{-\frac{\mathrm{i}E_S}{\hbar}t}\,\mathrm{e}^{-\frac{\Gamma_S}{2\hbar}t}, \tag{2.4}$$

where E_S is the total energy of particle; so $\omega_S = \frac{E_S}{\hbar}$ is the angular frequency and $\Gamma_S = \frac{\hbar}{\tau_S}$ is the width of the state. We can write the amplitude of the state K_2 in a similar fashion for the long lifetime. The intensity is

$$\begin{aligned} I_1(t) &= a_1(t)a_1^*(t) = a_1(0)a_1^*(0)\,\mathrm{e}^{-\frac{\Gamma_S}{\hbar}t} \\ &= I_1(0)\,\mathrm{e}^{-\frac{t}{\tau_S}}. \end{aligned} \tag{2.5}$$

Setting $\hbar = c = 1$ and considering a situation during which kaons are at rest we get that τ_S is the proper lifetime and $E_S = m_S$, the rest mass of the K_1 particle. Its amplitude is then

$$a_1(t) = a_1(0)\,\mathrm{e}^{-(\mathrm{i}m_S + \frac{\Gamma_S}{2})t}. \tag{2.6}$$

Similarly, for K_2,

$$a_2(t) = a_2(0)\,\mathrm{e}^{-(\mathrm{i}m_L + \frac{\Gamma_L}{2})t}. \tag{2.7}$$

From equation (2.2) we can write [18] the corresponding amplitudes of K_0 and $\overline{\mathrm{K}}_0$ as

$$a_0(t) = \frac{1}{\sqrt{2}}(a_1(t) + a_2(t)), \qquad \overline{a}_0(t) = \frac{1}{\sqrt{2}}(a_1(t) - a_2(t)), \tag{2.8}$$

and the intensities are equal to

$$I_0(t) = \frac{I_0(0)}{4}\left(\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} + 2\,\mathrm{e}^{-\frac{\Gamma_S + \Gamma_L}{2}t}\cos(\triangle m t)\right) \tag{2.9}$$

and

$$\overline{I}_0(t) = \frac{\overline{I}_0(0)}{4}\left(\mathrm{e}^{-\Gamma_S t} + \mathrm{e}^{-\Gamma_L t} - 2\,\mathrm{e}^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos(\triangle m t)\right). \tag{2.10}$$

Here $\triangle m = |m_L - m_S| \approx 3.52 \times 10^{-6}$ and $\triangle m \tau_S \approx 0.47$, so that K^0 and $\overline{\mathrm{K}}^0$ intensities *oscillate* with the frequency $|\triangle m|$.

This corresponds to the process called strangeness oscillation. We can explain it intuitively as follows: in the vacuum the disintegration of kaons is due to weak interactions, and the weak Hamiltonian controls and dominates the evolution. Therefore, the eigenstates of the 'free' (weak) Hamiltonian in vacuum are (in first approximation) the K_1 and K_2 kaons. In the presence of matter, strong interactions are present during the collisions between kaons and nuclei. They dominate the decay process and therefore K^0 and $\overline{\mathrm{K}}^0$ kaons are observed, and it is also possible to distinguish them experimentally because they possess different disintegration channels. This can be compared, if we develop the analogy with spin-1/2 systems, to situations in which the spin is measured along the Z direction while it undergoes a precession due to a magnetic field along the X direction between preparation and measurement. This is also analogue to what occurs when polarized light propagates in birefringent supports[3]. Because the preparation and measurement bases differ from the eigenbasis of the Hamiltonian that controls the free evolution, interference effects are likely to occur. This is the essence of strangeness oscillations. What is interesting is that if we compare their difference of mass (in convenient units) to the inverse of the lifetime of the K_1 kaon, we get a comparable result: $(m_S - m_L)\tau_S \approx 0,47$. Thanks to this relation and due to the fact that it was possible experimentally to carry out observations during a time comparable to the lifetime of the K_1 kaon, which is relatively long in comparison to other elementary particles, it was possible to observe strangeness oscillations experimentally.

Generation and regeneration are similar phenomena. If we produce (in matter, in the strong regime) K^0 particles, no $\overline{\mathrm{K}}^0$ particle is present, but if we wait (in the absence of matter) during a time long relatively to τ_S the lifetime of the K_1 kaon, the K_2 particle only has survived and the probability to find a $\overline{\mathrm{K}}^0$ particle is 0.5, so that $\overline{\mathrm{K}}^0$ particles were *generated*.

Regeneration is due to the fact that in the presence of matter, the $\overline{\mathrm{K}}^0$ particle disintegrates more quickly than the K^0 particle. Henceforth their respective amplitudes are not equal in modulus with as a consequence that $a_1(t) = \frac{1}{\sqrt{2}}(a_0(t) + \overline{a}_0(t))$ differs from zero. Consequently, even if we wait (in the absence of matter, in the weak regime) a time longer than the lifetime of the K_1 kaon, and that only the K_2 particle is present, the K_1 component is regenerated in the presence of matter.

CP violation is another interesting feature of the kaons phenomenology. It was discovered by Christenson *et al* [20]. CP violation means that CP symmetry is slightly violated (by a factor of 10^{-3}) by weak interactions so that the CP eigenstates K_1 and K_2 are not exact eigenstates of the decay interaction. Let us consider that $\mathrm{K}_S (S =$ short-lived) and $\mathrm{K}_L (L =$ long-lived) are the eigenstates of the decay interaction; they can be expressed as a superpositions of the K_1 and K_2 eigenstates. Then

$$\begin{aligned} |\mathrm{K}_L\rangle &= \frac{1}{\sqrt{1+|\epsilon|^2}}[\epsilon|\mathrm{K}_1\rangle + |\mathrm{K}_2\rangle] \\ &= \frac{1}{\sqrt{2(1+|\epsilon|^2)}}[(1+\epsilon)|\mathrm{K}^0\rangle - (1-\epsilon)|\overline{\mathrm{K}}^0\rangle], \end{aligned} \tag{2.11}$$

[3] This analogy is carefully developed in [10].

and

$$|\mathrm{K}_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}[|\mathrm{K}_1\rangle + \epsilon|\mathrm{K}_2\rangle] = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}[(1+\epsilon)|\mathrm{K}^0\rangle + (1-\epsilon)|\overline{\mathrm{K}}^0\rangle], \tag{2.12}$$

where $|\epsilon| \ll 1$ and ϵ does not have to be real. K_L and K_S are the eigenstates of the Hamiltonian for a mass-decay matrix [18, 19], i.e.

$$H = M - \frac{\mathrm{i}}{2}\Gamma \equiv \begin{pmatrix} M_{11} - \frac{\mathrm{i}}{2}\Gamma_{11} & M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12} \\ M_{21} - \frac{\mathrm{i}}{2}\Gamma_{21} & M_{22} - \frac{\mathrm{i}}{2}\Gamma_{22} \end{pmatrix}, \tag{2.13}$$

where M and Γ are individually Hermitian since they correspond to observables (mass and lifetime). The corresponding eigenvalues of the mass-decay matrix are equal to

$$m_L - \frac{\mathrm{i}}{2}\Gamma_L, \qquad m_S - \frac{\mathrm{i}}{2}\Gamma_S. \tag{2.14}$$

The CP violation was established by the observation that K_L decays not only via three-pion, which has natural CP parity, but also via the two-pion mode with a $|\epsilon|$ of order 10^{-3}, which is truly unexpected. The experimental value of ϵ is [21]:

$$|\epsilon| = (28.1 \pm 4.1) \times 10^{-4}, \qquad \arg(\epsilon) = 43.37. \tag{2.15}$$

3. Friedrichs's model and kaon phenomenology

In what follows, we shall identify the discrete modes of the Friedrichs model with the K_1 and K_2 states. This is our basic postulate according to which we can now make use of the Friedrichs model in order to establish a phenomenology for the kaonic behaviour. More precisely, we shall assume that

$$|\mathrm{K}_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \text{and} \qquad |\mathrm{K}_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{3.1}$$

Let us consider the solution of the two-level Friedrichs model Schrödinger equation (1.6). According to this equation, the state is at time t superposition of two components that correspond to the two (complex) eigenvalues of the effective Hamiltonian. In order to avoid confusion, we shall use different parameters when we deal with the 'real' kaons that are associated with the experimental data and when we deal with the 'theoretic' ones in the framework of the Friedrichs model.

– The masses m_S and m_L and the lifetime τ_S and τ_L will remain attributed to the real objects.

– The parameters ω_1, ω_2, λ_1, λ_2, ω_+ and ω_- will refer to the theoretic quantities.

3.1. Solutions for $v(\omega) = \mathrm{e}^{-\alpha\omega^2/2}$, $\alpha > 0$, $\alpha \to 0$

3.1.1. Case $\omega \in]-\infty, +\infty[$. If we substitute $v(\omega) = \mathrm{e}^{-\alpha\omega^2/2}$ in equations (1.11) and integrate from $-\infty$ to ∞ we obtain

$$\mathrm{i}\frac{\partial f_1(t)}{\partial t} = \omega_1 f_1(t) - \mathrm{i}\lambda_1\sqrt{\frac{\pi}{\alpha}}[\lambda_1 f_1(t) + \lambda_2 f_2(t)] * \mathrm{e}^{-\frac{t^2}{4\alpha}}, \tag{3.2}$$

where we used the convolution relation, i.e.

$$\frac{\partial y(t)}{\partial t} = f(t) + \int_0^t k(t-u)y(u)\,\mathrm{d}u = f(t) + k(t) * y(t). \tag{3.3}$$

Then the Laplace transformation of the above equation is

$$sY(s) - y(0) = F(s) + K(s)Y(s). \tag{3.4}$$

Thus we write the Laplace transformation of equation (3.2) as

$$\mathrm{i}(sF_1(s) - f_1(0)) = \omega_1 F_1(s) - \mathrm{i}\pi\lambda_1[\lambda_1 F_1(s) + \lambda_2 F_2(s)]\,\mathrm{e}^{\alpha s^2}\,\mathrm{Erfc}(\sqrt{\alpha}s), \tag{3.5}$$

where

$$\mathrm{Erfc}(x) = 1 - \mathrm{Erf}(x) = 1 - \frac{2}{\sqrt{\pi}}\int_0^x \mathrm{e}^{-y^2}\,\mathrm{d}y. \tag{3.6}$$

Expanding Erfc and exponential functions (this corresponds to the Markovian or Wigner–Weisskopf regime) we obtain

$$\mathrm{i}(sF_1(s) - f_1(0)) = \omega_1 F_1(s) - \mathrm{i}\pi\lambda_1[\lambda_1 F_1(s) + \lambda_2 F_2(s)]\left(1 - 2\sqrt{\frac{\alpha}{\pi}}s\right) + O(\alpha). \tag{3.7}$$

Now, the inverse Laplace transformation yields

$$\mathrm{i}\big(1 - 2\sqrt{\pi\alpha}\lambda_1^2\big)\frac{\partial f_1(t)}{\partial t} = \omega_1 f_1(t) - \mathrm{i}\pi\lambda_1[\lambda_1 f_1(t) + \lambda_2 f_2(t)] + 2\mathrm{i}\lambda_1\lambda_2\sqrt{\pi\alpha}\frac{\partial f_2(t)}{\partial t}. \tag{3.8}$$

We can obtain the same relation for f_2. Then we can easily obtain

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix} = \begin{pmatrix} \frac{\omega_1 - \mathrm{i}\pi\lambda_1^2}{1-2\sqrt{\pi\alpha}\lambda_1^2} & \lambda_1\lambda_2\big(-\frac{\mathrm{i}\pi}{(1-2\sqrt{\pi\alpha}\lambda_1^2)} + \frac{2\sqrt{\pi\alpha}\omega_2}{1-2\sqrt{\pi\alpha}(\lambda_1^2+\lambda_2^2)}\big) \\ \lambda_1\lambda_2\big(-\frac{\mathrm{i}\pi}{(1-2\sqrt{\pi\alpha}\lambda_2^2)} + \frac{2\sqrt{\pi\alpha}\omega_1}{1-2\sqrt{\pi\alpha}(\lambda_1^2+\lambda_2^2)}\big) & \frac{\omega_2 - \mathrm{i}\pi\lambda_2^2}{1-2\sqrt{\pi\alpha}\lambda_2^2} \end{pmatrix} \times \begin{pmatrix} f_1(t) \\ f_2(t) \end{pmatrix}, \tag{3.9}$$

in which we neglect the $O(\lambda^4)$ contributions. The eigenvalues of the above effective Hamiltonian, here denoted by H_{eff}, are

$$\omega_+ = \frac{\omega_1 - \mathrm{i}\pi\lambda_1^2}{1 - 2\sqrt{\pi\alpha}\lambda_1^2} + O(\lambda^4) \approx \big(\omega_1 - \mathrm{i}\pi\lambda_1^2\big)\big(1 + 2\sqrt{\pi\alpha}\lambda_1^2 + \cdots\big) \approx \big(1 + 2\sqrt{\pi\alpha}\lambda_1^2\big)\omega_1 - \mathrm{i}\pi\lambda_1^2, \tag{3.10}$$

and

$$\omega_- \approx \big(1 + 2\sqrt{\pi\alpha}\lambda_2^2\big)\omega_2 - \mathrm{i}\pi\lambda_2^2. \tag{3.11}$$

In this approximation the eigenvectors of the effective Hamiltonian are obtained as follows:

$$|f_+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \text{and} \qquad |f_-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{3.12}$$

Comparing the eigenvalues in equations (3.10) and (3.11) with the equations in (2.14) we obtain

$$\begin{aligned} 2\pi\lambda_1^2 &= \Gamma_S, \qquad & \omega_1 &= \frac{m_S}{1 + 2\sqrt{\pi\alpha}\Gamma_S} \approx m_S(1 - 2\sqrt{\pi\alpha}\Gamma_S), \\ 2\pi\lambda_2^2 &= \Gamma_L, \qquad & \omega_2 &= \frac{m_L}{1 + 2\sqrt{\pi\alpha}\Gamma_L} \approx m_L(1 - 2\sqrt{\pi\alpha}\Gamma_L). \end{aligned} \tag{3.13}$$

CP violation. Let us study in this case the CP violation. The Friedrichs model allows us to estimate the value of ϵ. If the effective Hamiltonian (equation (3.9)) acts on $|\mathrm{K}_L\rangle$ vector states

(equation (2.11)) as an eigenstate corresponding to the eigenvalue $\omega_- = \big(1+2\sqrt{\pi\alpha}\lambda_2^2\big)\omega_2 - \mathrm{i}\pi\lambda_2^2$ we must impose (as the sign of ϵ is arbitrary), that $H_{\mathrm{eff}}\binom{-\epsilon}{1} = \omega_-\binom{-\epsilon}{1}$, from which we obtain straightforwardly that, at the dominating order,

$$\epsilon = \frac{-\lambda_1\lambda_2\big(-\frac{\mathrm{i}\pi}{(1-2\sqrt{\pi\alpha}\lambda_1^2)} + \frac{2\sqrt{\pi\alpha}\omega_2}{1-2\sqrt{\pi\alpha}(\lambda_1^2+\lambda_2^2)}\big)}{\big[\big(1+2\sqrt{\pi\alpha}\lambda_2^2\big)\omega_2 - \mathrm{i}\pi\lambda_2^2\big] - \big[\big(1+2\sqrt{\pi\alpha}\lambda_1^2\big)\omega_1 - \mathrm{i}\pi\lambda_1^2\big]}, \tag{3.14}$$

and if we expand and neglect $O(\lambda^4)$, we obtain

$$\epsilon \approx \frac{-\lambda_1\lambda_2(-\mathrm{i}\pi + 2\sqrt{\pi\alpha}\omega_2)}{\big(1+2\sqrt{\pi\alpha}\lambda_1^2\big)\omega_2 - \big(1+2\sqrt{\pi\alpha}\lambda_2^2\big)\omega_1 - \mathrm{i}\pi\big(\lambda_2^2-\lambda_1^2\big)}. \tag{3.15}$$

If we replace λs and ωs by their corresponding values from equation (3.13) we have

$$\epsilon \approx \frac{\frac{\mathrm{i}}{2}\sqrt{\Gamma_L\Gamma_S}\big(1+2\mathrm{i}\sqrt{\frac{\alpha}{\pi}}m_S\big)}{(m_L-m_S) - \frac{\mathrm{i}}{2}(\Gamma_L-\Gamma_S)}. \tag{3.16}$$

In the zeroth approximation of α we obtain thus

$$\epsilon \approx \sqrt{(1.82\times10^{-3})/2}\,\mathrm{e}^{\mathrm{i}(43.37)^\circ}, \tag{3.17}$$

which shows that our estimation of the modulus of ϵ is $\sim$14 times greater than its experimental value while the estimated phase is correct. Now, in the case $\alpha \neq 0$, ϵ is given as

$$\epsilon \approx \sqrt{(1.82\times10^{-3})/2}\,\mathrm{e}^{\mathrm{i}(43.37)^\circ}\left(1+2\mathrm{i}\sqrt{\frac{\alpha}{\pi}}m_S\right). \tag{3.18}$$

We see that $\alpha > 0$ both changes the argument of ϵ and increases its modulus. Henceforth, a Gaussian test function in $]-\infty,\infty[$ is not a good choice if we aim at improving the fit with the experimental CP violation.

3.1.2. Case $\omega \in [0,+\infty[$. If we substitute $v(\omega) = \mathrm{e}^{-\alpha\omega^2/2}$ in equations (1.11) and integrate from 0 to ∞ we obtain

$$\mathrm{i}\frac{\partial f_1(t)}{\partial t} = \omega_1 f_1(t) - \mathrm{i}\frac{\lambda_1}{2}\sqrt{\frac{\pi}{\alpha}}[\lambda_1 f_1(t) + \lambda_2 f_2(t)] * \mathrm{e}^{-\frac{t^2}{4\alpha}}\,\mathrm{Erfc}\left(\frac{\mathrm{i}t}{2\sqrt{\alpha}}\right). \tag{3.19}$$

The Laplace transformation and the expansion in α of the above equation (3.19) lead to

$$\mathrm{i}(sF_1(s) - f_1(0)) = \omega_1 F_1(s) - \frac{\mathrm{i}\pi\lambda_1}{2}[\lambda_1 F_1(s) + \lambda_2 F_2(s)]$$
$$\times\left[\left(1-\frac{2\mathrm{i}}{\pi}\right) + 2(-1+\mathrm{i})\sqrt{\frac{\alpha}{\pi}}s\right] + O(\alpha). \tag{3.20}$$

Now, the inverse Laplace transformation yields

$$\mathrm{i}\big(1-(1-\mathrm{i})\sqrt{\pi\alpha}\lambda_1^2\big)\frac{\partial f_1(t)}{\partial t} = \omega_1 f_1(t) - \frac{(\mathrm{i}\pi+2)\lambda_1}{2}$$
$$\times[\lambda_1 f_1(t) + \lambda_2 f_2(t)] + (1+\mathrm{i})\lambda_1\lambda_2\sqrt{\pi\alpha}\frac{\partial f_2(t)}{\partial t}. \tag{3.21}$$

We can obtain the same relation for f_2. Then we can easily obtain

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix} f_1(t) \\ f_2(t)\end{pmatrix}$$
$$= \begin{pmatrix} \frac{\omega_1-(\frac{\mathrm{i}\pi}{2}+1)\lambda_1^2}{1-(1-\mathrm{i})\sqrt{\pi\alpha}\lambda_1^2} & \lambda_1\lambda_2\big(-\frac{(\mathrm{i}\pi+2)}{2(1-(1-\mathrm{i})\sqrt{\pi\alpha}\lambda_1^2)} + \frac{(1+\mathrm{i})\sqrt{\pi\alpha}\omega_2}{1-(1-\mathrm{i})\sqrt{\pi\alpha}(\lambda_1^2+\lambda_2^2)}\big) \\ \lambda_1\lambda_2\big(-\frac{(\mathrm{i}\pi+2)}{2(1-(1-\mathrm{i})\sqrt{\pi\alpha}\lambda_2^2)} + \frac{(1+\mathrm{i})\sqrt{\pi\alpha}\omega_1}{1-(1-\mathrm{i})\sqrt{\pi\alpha}(\lambda_1^2+\lambda_2^2)}\big) & \frac{\omega_2-(\frac{\mathrm{i}\pi}{2}+1)\lambda_2^2}{1-(1-\mathrm{i})\sqrt{\pi\alpha}\lambda_2^2}\end{pmatrix}$$
$$\times\begin{pmatrix} f_1(t) \\ f_2(t)\end{pmatrix}, \tag{3.22}$$

in which we neglected the $O(\lambda^4)$ contributions. Obtaining the eigenvalues of the above effective Hamiltonian and comparing with the equations in (2.14) we get

$$\begin{aligned} \lambda_1^2 &\approx \frac{\Gamma_S}{\pi}\left(1-2m_S\sqrt{\frac{\alpha}{\pi}}\right), \qquad & \omega_1 &\approx m_S+\Gamma_S-6m_S\Gamma_S\sqrt{\frac{\alpha}{\pi}}, \\ \lambda_2^2 &\approx \frac{\Gamma_L}{\pi}\left(1-2m_L\sqrt{\frac{\alpha}{\pi}}\right), \qquad & \omega_2 &\approx m_L+\Gamma_L-6m_L\Gamma_L\sqrt{\frac{\alpha}{\pi}}. \end{aligned} \tag{3.23}$$

C P violation. Once again, let us estimate the value of ϵ. We obtain

$$\begin{aligned} \epsilon &\approx \frac{\mathrm{i}\sqrt{\Gamma_L\Gamma_S}\left[\left(\frac{1}{2}-\mathrm{i}\frac{1}{\pi}\right)+\left(-2+\mathrm{i}\frac{\pi+2}{\pi}\right)m_S\sqrt{\frac{\alpha}{\pi}}\right]}{(m_L-m_S)-\frac{\mathrm{i}}{2}(\Gamma_L-\Gamma_S)} \\ &\approx \sqrt{2(1.82\times10^{-3})}\,\mathrm{e}^{\mathrm{i}(43.37)^\circ}\left[\left(\frac{1}{2}-\mathrm{i}\frac{1}{\pi}\right)+\left(-2+\mathrm{i}\frac{\pi+2}{\pi}\right)m_S\sqrt{\frac{\alpha}{\pi}}\right]. \end{aligned} \tag{3.24}$$

We see that if $m_S\sqrt{\frac{\alpha}{\pi}}=\frac{1}{2+\pi}$ the imaginary part in the bracket of the above equation is zero and the real part is equal to 0.111, which corresponds to the estimation

$$\epsilon = 6.69\times10^{-3}\,\mathrm{e}^{\mathrm{i}(43.37)^\circ}. \tag{3.25}$$

So, in this case, $|\epsilon| = 6.69\times10^{-3}$ which is only $\sim$ three times greater than the experimental value while the estimated phase is correct.

4. Discussion of other approaches

We present here a heuristic discussion of other recent approaches to the decay phenomena in quantum mechanics and a comparison with the Friedrichs model. Our approach in the above sections considers the decay, oscillation, regeneration and $C\,P$ violation of kaons in the Hilbert space. On the other hand, the open systems approach aims, briefly speaking, to study some basic questions namely decoherence, Bell inequality, nonlocality, etc [10, 13, 14, 17, 22, 23].

The key ingredient for deriving irreversible in time evolution laws from the unitary Schrödinger evolution is indeed, in open systems approach, to focus on subsystems of a very large system (system plus environment). The role of the environment is played in the Friedrichs model by continuous degree of freedom, while the subsystem is a discrete (two-level in our case) system. As we have shown, these ingredients (discrete system coupled to a continuum) suffice in order to be able to derive a non-unitary master equation for the two-level system. It is worth noting that this approach in which the environment is coupled to the subsystem is very general in quantum physics. It is for instance the approach followed in order to derive master equations [14, 13], or to solve the measurement problem in the so-called decoherence approach [24] and it led to interesting treatments of the general Zeno paradox in the sense of [8, 9].

In the usual formulation of the Friedrichs model, the border line between the system and environment is ill defined because the Hilbert spaces associated with those degrees of freedom are not the tensorial product of their respective Hilbert spaces but are rather their direct sum. Nevertheless it is possible, as we shall see below, to imbed the direct sum of the Hilbert spaces associated with the discrete and continuous degrees of freedom into a larger space in which those subspaces (tensorially) factorize, and to formulate an equivalent Hamiltonian dynamics that contains as a special subset of solutions all the solutions of the original model. Such a framework is also useful and necessary, as we shall show, in order to compare our model with other approaches of open, dissipative, noisy dynamics that have recently been proposed to describe kaon phenomenology and possible new experimental tests on entangled kaonic pairs.

This modified Friedrichs model can be explained heuristically as follows. Instead of representing the state of the system at time t by a direct sum of the Hilbert spaces associated with the discrete and continuous degrees of freedom, we imbed it into the tensorial products of a three-dimensional Hilbert space $\mathbb{C}^3$ (that corresponds to the two discrete levels plus their decay product) and of a Fock space; $\mathbb{C}^3 \bigotimes \mathcal{H}_{\text{photon}}$,

$$\psi_{\text{kaon}} = \begin{pmatrix} f_0 \\ f_1 \\ f_2 \end{pmatrix} \qquad \text{and} \qquad \psi_{\text{photon}} = \begin{pmatrix} f^0 \\ f^1(\omega') \\ f^2(\omega', \omega'') \\ \dots \end{pmatrix}$$

and the state is given by

$$\Psi_{0,1,2,\omega^i} = \begin{pmatrix} f_0 \\ 0 \\ 0 \end{pmatrix} \bigotimes \begin{pmatrix} 0 \\ f^1(\omega') \\ f^2(\omega', \omega'') \\ . \\ . \\ . \\ f^2(\omega', \omega'', \dots, \omega^{(n)}) \\ \dots \end{pmatrix} + \begin{pmatrix} 0 \\ f_1 \\ f_2 \end{pmatrix} \bigotimes \begin{pmatrix} f^0 \\ 0 \\ 0 \\ . \\ . \\ . \end{pmatrix}, \tag{4.1}$$

where f_0 represents the amplitude of a new discrete state $|0\rangle$ that is assumed to contain the 'decay products' resulting from the disintegration of the two discrete kaonic states $|1\rangle$ and $|2\rangle$; besides, $f^n(\omega', \omega'', \dots, \omega^{(n)})$ $(n = 1, 2, \dots)$ represents the amplitude of the n environment particles.

Now that we defined our representation of the state of the system, we can define the free Hamiltonian:

$$H_{\text{free}} = \begin{pmatrix} \omega_0 & 0 & 0 \\ 0 & \omega_1 & 0 \\ 0 & 0 & \omega_2 \end{pmatrix} \otimes Id.^{\omega} + Id.^{0,1,2} \otimes \omega a^{\dagger}.a. \tag{4.2}$$

The first part of it represents the energies of the discrete modes, while the second one contains the energies of the excited modes. Here the operators $a^{\dagger}.a$ count the number of excitations in the mode ω.

The interaction Hamiltonian, H_{int}, is equal to

$$H_{\text{int}} = \begin{pmatrix} 0 & \lambda_1 v(\omega) & \lambda_2 v(\omega) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \otimes a^{\dagger} + \begin{pmatrix} 0 & 0 & 0 \\ \lambda_1 v^*(\omega) & 0 & 0 \\ \lambda_2 v^*(\omega) & 0 & 0 \end{pmatrix} \otimes a. \tag{4.3}$$

In analogy with quantum optics, this interaction represents the decay of the kaonic 'excited' states (1 and 2) to the 'ground' state (0), with excitation of a mode of energy (ω) while by unitarity the inverse process is also possible (diminution of the energy of a continuous mode by one quantum of energy ω (here $\hbar = 1$), and repopulation of the discrete states $|1\rangle$ and $|2\rangle$). If the initial state is such that no continuous mode is excited ($f^i(\omega', \omega'', \dots, \omega^{(i)}, t = 0) = 0 \, \forall i > 0$), then, the dynamics of the state $\Psi_{0,1,2,\omega}(t)$ is considerably simplified because there will never occur more than one excitation.

In that case $f_1(t)$, $f_2(t)$ and $f^1(\omega, t)$ obey a closed system of three equations

$$\omega_1 f_1(t) + \lambda_1 \int \mathrm{d}\omega \, v^*(\omega) f^1(\omega, t) = \mathrm{i} \frac{\partial f_1(t)}{\partial t}, \tag{4.4}$$

$$\omega_2 f_2(t) + \lambda_2 \int \mathrm{d}\omega \, v^*(\omega) f^1(\omega, t) = \mathrm{i} \frac{\partial f_2(t)}{\partial t}, \tag{4.5}$$

and

$$\lambda_1 v(\omega) f_1(t) + \lambda_2 v(\omega) f_2(t) + \omega f^1(\omega, t) = \mathrm{i}\frac{\partial f^1(\omega, t)}{\partial t}, \tag{4.6}$$

where we used $\langle v(\omega), f^1(\omega)\rangle = \int \mathrm{d}\omega\, v^*(\omega) f^1(\omega)$, and the components f_0 and f^0 remain unaffected on the evolution so we take them equal to 1, all the other modes are zero.

After identification of $f^1(\omega, t)$ and our previous $g(\omega, t)$, we recover a system of equations of evolution that is rigorously identical to the system of equations (1.7), (1.8) and (1.9) derived in the framework of the Friedrichs model.

At this level, we can compare the Friedrichs model with other models where a system interacts with its environment, which leads to a non-unitary evolution for the reduced system. Quite an amount of recent literature [10, 11, 14], theoretical and phenomenological, deals precisely with the possibility of treating kaon properties by making them an open system. In those dissipative models, the evolution law can be brought to the Lindblad form (this form is standard provided we assume that the evolution law is completely positive, not merely positive, an hypothesis which can be justified, to some extent, on physical grounds [14]). The Markovian Lindblad evolution has the following form:

$$\frac{\partial}{\partial t}\rho = -\mathrm{i}H_{\mathrm{eff}}\rho + \mathrm{i}\rho H_{\mathrm{eff}}^{\dagger} - D(\rho), \tag{4.7}$$

where H_{eff} is a non-necessary Hermitian Hamiltonian, while $D(\rho)$ is the so-called dissipator (see e.g. [11, 13] for its precise description) of which the main effect is that it induces a loss of coherence of the reduced system. This is not the case with the Hamiltonian part H_{eff} of the evolution operator as shown by direct computation: the von Neumann entropy of the reduced system $\rho_N = \rho/(\mathrm{Tr}(\rho))$, which is by definition equal to $S(\rho_N(t)) = -\mathrm{Tr}(\rho_N(t)\log_2 \rho_N(t))$ is constant in time whenever the dissipator $D(\rho)$ is identically equal to zero throughout time. This means among others that in this case pure states remain pure states (up to a global decay), which is effectively the case in our model where the dissipator is identically equal to zero throughout time as can be seen from the Markovian limits (3.9) and (3.22) (the Wigner–Weisskopf regime) of equations (1.11) and (1.12). However, another result may be obtained if we consider a weak coupling limit of the evolution of a kaon system state tensorized with an equilibrium state of an infinite environment [27]. This is quite different from our approach and out of the scope of this paper which makes the weak coupling limit in the frame of the Hilbert space. Here we considered only the process of emission and absorption of one particle of the environment.

It is worth noting that properties that make kaonic phenomenology so interesting and attractive such as oscillations, generation and regeneration are a manifestation of the superposition principle, which is of application precisely because pure states remain pure throughout time.

Remark. The coherence between the decay products $|0\rangle$ on one side and the space spanned by the kaonic modes $|1\rangle$ and $|2\rangle$ on the other side is not preserved under the partial trace. In fact, this comes out from the computation of the partial trace over the biorthogonal decomposition of the full state:

$$\Psi_{0,1,2,\omega^i}(t) = \begin{pmatrix}1\\0\\0\end{pmatrix} \bigotimes \begin{pmatrix}0\\ f^1(\omega_1, t)\\0\\.\\.\\.\end{pmatrix} + \begin{pmatrix}0\\ f_1(t)\\ f_2(t)\end{pmatrix} \bigotimes \begin{pmatrix}1\\0\\0\\.\\.\\.\end{pmatrix}. \tag{4.8}$$

Recall that if $|\Psi\rangle^{AB} = \sum_{i,j} \alpha_{ij}|i\rangle^A \otimes |j\rangle^B$ then

$$\mathrm{Tr}_A(|\Psi\rangle^{AB\ AB}\langle\Psi|) = \sum_{i,j,j'} \alpha_{ij}\alpha_{ij'}|j\rangle^B \otimes {}^B\langle j'| \tag{4.9}$$

and

$$\mathrm{Tr}_B(|\Psi\rangle^{AB\ AB}\langle\Psi|) = \sum_{i,i',j} \alpha_{ij}\alpha_{ij'}|i\rangle^A \otimes {}^A\langle i'|. \tag{4.10}$$

Then, taking a partial trace of this state over continuous degrees of freedom it is easy to check that the reduced density matrix of the discrete (tripartite) degrees of freedom is equal to

$$\rho^{\mathrm{kaons}} = \begin{pmatrix} \|f^1\|^2 & 0 & 0 \\ 0 & |f_1|^2 & f_1 f_2^* \\ 0 & f_1^* f_2 & |f_2|^2 \end{pmatrix}, \tag{4.11}$$

where $\|f^1\|^2 = \int |f^1(\omega)|^2 \,\mathrm{d}\omega$ and $\|f^1\|^2 + |f_1|^2 + |f_2|^2 = 1$. This is clearly the incoherent sum of the decay products and a pure state that is coherent superposition of the K_1 and K_2 modes.

5. Concluding remarks

We have shown that the framework of the Friedrichs model is relevant in order to grasp, despite its simplicity, essential features of kaons decay. This model allows us to describe complex temporal evolutions (such as kaonic oscillations, generation and regeneration) and to simulate at least qualitatively CP violation. We also recover the experimental value of the phase, 43.37° as a results of equations (3.16), (3.17) and (3.25).

The measurement problem suggests that two regimes characterize the temporal evolution of a quantum system: a continuous, unitary evolution in the absence of measurement, and a sudden, irreversible in time evolution during the measurement process (quantum jump). In the present paper, we studied an approach in which the evolution of a two-level system coupled to a continuum is continuous in time but possesses both a unitary and a non-unitary components.

We showed that in the framework of the Friedrichs model the main feature that is responsible for the derivation of an irreversible in time master equation for the discrete system is the energy continuum. We also showed that the Friedrichs model is relevant in order to describe complex temporal evolutions (such as kaonic oscillations, generation and regeneration) and to simulate at least qualitatively CP violation.

In section 4, we discussed the Lindbladian approach to decay problem using a Fock space formulation for the Friedrichs model.

It is out of the scope of the present paper, but it would be very interesting to study the properties of the Friedrichs model and of kaonic oscillations in terms of the time operator approach. This can be done for the one-level Friedrichs model [25, 26], but higher level systems present more subtle and involved temporal behaviour [9] so that it is worth studying the time operator in this context.

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